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What you'll Learn About

- How to find the derivative of inverse functions

A) $y = \arcsin(x^2)$

u = ratio

$$y' = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

B) $y = \arccos\left(\frac{1}{x}\right)$

$$y = \arccos(x^{-1})$$

$$y' = \frac{-1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \cdot -x^{-2} = \frac{x^{-2}}{\sqrt{1-\frac{1}{x^2}}}$$

$$= \frac{1}{x^2\sqrt{1-\frac{1}{x^2}}} = \frac{1}{\frac{x^2}{1}\sqrt{\frac{x^2-1}{x^2}}} = \frac{1}{|x|\sqrt{x^2-1}}$$

C) $y = x^2\arccos(\sin x)$

$$y' = x^2 \cdot \left(\frac{-1}{\sqrt{1-\sin^2 x}}\right)(\cos x) + \arccos(\sin x)(2x)$$

$$y' = \frac{-x^2\cos x}{\sqrt{\cos^2 x}} + 2x\arccos(\sin x)$$

D) $y = x\sqrt{1-x^2} + \arctan\sqrt[3]{x}$

$$y = x(1-x^2)^{1/2} + \arctan(x^{1/3})$$

$$y' = x\left\{\frac{1}{2}(1-x^2)^{-1/2}\right\}\cdot(-2x) + (1-x^2)^{1/2} + \frac{1}{1+(x^{1/3})^2}\cdot\frac{1}{3}x^{-2/3}$$

$$y' = \frac{-x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2} + \frac{1}{3x^{2/3}(1+x^{2/3})}$$

E) $f(x) = \operatorname{arccot}(5x^3 - \sin x)$

$$f'(x) = \frac{-1}{1+(5x^3-\sin x)^2}\cdot(15x^2-\cos x)$$

$$= \frac{-(15x^2-\cos x)}{1+(5x^3-\sin x)^2}$$